Signals and codes homework assignments

This document describes homework assignments in course signals and codes, taught at faculty of transportation sciences CTU in Prague.

1 General information

1.1 Communication

- The assignments are assigned and collected electronically. You could bring them on paper for review or consultation but to be "evaluated" they should be sent by an email!
- Ideal form of the assignment is written on paper (in script that I could read) and scanned to ONE document (pdf) of reasonable size. Preferably all HWs in one document, but in case of different dates of hand out or corrections just sent the actual (corrected / new) parts.
- Scanned document apart from HW assignment and its solution shall contain: course / year / student / version / date / page number information!

1.2 Due dates

The due date of assignments is the end of the semester or exam date at the latest.

1.3 Scoring

All assignments together will be evaluated to score 0-10 points (point distribution among the assignments will be set once they are all sent out to students (10 Hws = 1 point per each, less than 10 HWs = some assignment 2 points …).

2 Homework assignments list

This chapter just contains the current list of the assignment contained in the document.

2.1 Signals assignments

- EX: Modulation vs transmission speeds
- EX: Power and energy
- EX: Signal transformations
- EX: Spectrum of sinusoid combination
- EX: Spectrum of AM modulation
- EX: Fourier series – sine cubed signal
- EX: Spectrum and complex numbers
- EX: Fourier series – triangle signal

2.2 Codes assignments

3 Signals assignments

3.1 EX: Modulation vs transmission speeds

Consider signal limited in frequency by $f_{min} = 10\ kHz$ and $f_{max} = 50\ kHz$ find maximum modulation speed [baud/s] that could be used to convey information through this channel.
What is the maximum transmission speed [bits/s] in error free environment without attenuation?
Discuss the real maximum transmission speed (obtained by Shannon's information theorem) over the "air interface", where SNR is 25 dB.

3.2 EX: Power and energy

Compute energy and power and decide whether the signals are energy or power signals for following signals:

- $x(t) = \begin{cases} 8\ for\ |t| < 5 \\ 0\ otherwise \end{cases}$
- $x[n] = j$
- $x(t) = \begin{cases} 3e^{at}\ for\ t > 0 \\ 0\ otherwise \end{cases}$, for all real a values!
- $x(t) = 2\cos(2t)$

3.3 EX: Signal transformations

Construct odd and even signals $x_o(t)$ resp. $x_e(t)$ for signal $x(t)$. Graphically prove that $x(t) = x_e(t) + x_o(t)$.

$$x(t) = \begin{cases} 0\ for\ t > 2\ and\ t < -1 \\ -2\ for\ t \in \langle -1,0) \\ 3t - 2\ for\ t \in \langle 0,1) \\ -t + 2\ for\ t \in \langle 1,2\rangle \end{cases}$$

3.4 EX: Spectrum of sinusoid combination

A signal composed of sinusoids is given by the equation $x(t) = 3\cos(50\pi t - \pi/8) - 5\cos(150\pi t + \pi/6)$

a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.
b) Is $x(t)$ periodic? If so, what is the period? Which harmonics are present?
c) Now consider a new signal $y(t) = x(t)+7\cos(160\pi t - \pi/3)$. How is the spectrum changed? Is $y(t)$ periodic? If so, what is the period?
d) Finally, consider another new signal $w(t) = x(t)+\cos(5\sqrt{2}\,\pi t+\pi/3)$. How is the spectrum changed? Is w(t) periodic? If so, what is the period? If not, why not?

3.5 EX: Spectrum of AM modulation

The signal x(t) is formed from the signal v(t) by AM modulation. Assume that $v(t) = 3 + 3\cos(5t + \pi/3)$ and that $x(t) = v(t)\cos(20t)$.

a) Find an additive combination (using complex exponentials) for $x(t)$. Use the inverse Euler formula rather than a trigonometric identity
b) Count the number of frequency components in the spectrum.
c) What is the highest frequency contained in $x(t)$?
a) Draw the spectrum for $v(t)$.
b) Draw the spectrum for $x(t)$.

3.6 EX: Fourier series – sine cubed signal

Use the Fourier integral to determine all the Fourier series coefficients of the "sine-cubed" signal. In other words, evaluate the integral

$$a_k = \frac{1}{T_0}\int_0^{T_0} \sin^3(3\pi t)e^{-j(2\pi/T_0)kt}\,dt$$

for all k.

Hints: find the period first, so that the integration interval is known. In addition, you might find it easier to convert the $\sin^3(\cdot)$ function to exponential form (via the inverse Euler formula for sin(·)) before doing the Fourier integral on each of four different terms. Then **invoke the orthogonality property** on each integral.

3.7 EX: Spectrum and complex numbers

Make a sketch of the spectrum of the signal defined as:

$$x(t) = \sum_{k=-3}^{k=3} \frac{1}{1+jk} e^{jkt}$$

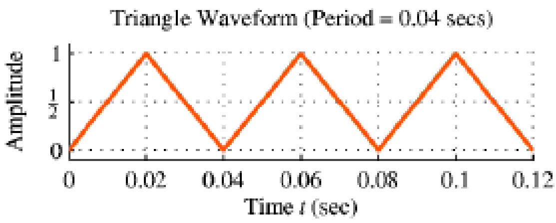
Use conversion from Cartesian form of the complex number to polar form to find out complex amplitudes. For negative part of the spectrum you can use complex conjugate property.

$$re^{j\theta} = r\cos\theta + j\sin\theta$$

$$z = a + jb \rightarrow |z| = r = \sqrt{a^2 + b^2} \; and \; \theta = \tan^{-1}\frac{b}{a}$$

3.8 EX: Fourier series – triangle signal

Derive the formula for the Fourier Series coefficients of the triangle wave defined by $x(t)$ below in the interval $0 < t < T_0$. Use integration by parts to manipulate the integrands which contain terms of the form $te^{-j(2\pi/T)kt}$.



$$x(t) = \begin{cases} 2t/T_0 & \text{for } 0 \le t < \frac{1}{2}T_0 \\ 2(T_0 - t)/T_0 & \text{for } \frac{1}{2}T_0 \le t < T_0 \end{cases}$$

Make a plot of the spectrum for the triangle wave. Use the complex amplitudes and assume that $f_0 = 25$ *Hz*.
For the N = 3 approximation of the triangle wave, derive the mathematical formula for the sinusoids.