ARITHMETIC CODING

- re-normalisation

A

0/3 1/3 2/3 3/3

0 1/3 2/3 1

we will allways use $\langle 0,1)$ to assign intervals for letters and scale afterwards

DECODING

message: $0.001011011 = \frac{91}{512}$ 0.177...

(1) 0/3 1/3 2/3 3/3 → A

$\left(\frac{91}{512} - \text{lower bound of interval}\right) / \text{length of interval}$

$\Rightarrow$ a number from $\langle 0,1) = \frac{273}{512}$ 0.53...

$\frac{91}{512} / \frac{1}{3} = \frac{3 \cdot 91}{512} = \frac{273}{512}$

(2) 0/3 1/3 2/3 3/3 → B

$\left(\frac{273}{512} - \frac{1}{3}\right) \cdot 3 = \frac{3 \cdot 273 - 512}{512} = \frac{307}{512}$

ABBCBA $\boxed{EOM}$

ABBCBAAAAA....

A ... 0

B ... 1/3

C ... 2/3

Ex: Hamming (3,1) ... repetition code

$$G_{sys} = [1 \; 1 \; 1]$$

$$P = [1\,1] \Rightarrow P^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$H_{sys} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{array}{l} \vec{w} = (000) \\ \vec{e} = (00\underline{1}) \end{array}\right\} w' = (001)$$

$$w' \cdot H^T = (001) \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$w' \cdot H^T = (101) \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$n = 2^m - 1$$

HAMMING CODES

Construction:

a) from H (7,4)

$$H = \underbrace{\begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}}_{n \text{ columns}} \Bigg\} m \text{ rows}$$

- columns are distinct
- represent numbers from $\underline{1}$ to $\underline{n}$

$$H_{sys} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

b) using parity bit assignment
non-systematic code
construct a parity bit assignment and
copy it to systematic G afterwards

1	2	3	4	5	6	7
p_0	p_1	d_0	p_2	d_1	d_2	d_3

p_0	X		X		X		X
p_1		X	X			X	X
p_2				X	X	X	X

position 1 ⇒ ... 1 (2^1)
position 2 ⇒ .. 1 . (2^2)
position 4 ⇒ . 1 .. (2^3)

	$p_2\,p_1\,p_0$
1	... 001
2	... 010
3	... 011
4	... 100
5	... 101
6	... 110
7	... 111

$$G = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}}_{n} \Bigg\} k$$

$$\vec{u} \cdot \vec{G} = \vec{w}$$

$$\Rightarrow \begin{array}{l} p_0 = d_0 \oplus d_1 \oplus d_3 \\ p_1 = d_0 \oplus d_2 \oplus d_3 \\ p_2 = d_1 \oplus d_2 \oplus d_3 \end{array}$$

Construction:

$\boxed{w(x) = u(x) \cdot g(x) \ \underline{\mathrm{mod}}\ x^n \oplus 1}$

n ... code length, k ... number of data bits

m ... number of parity bits

$g(x)$: generator polynomial

$\deg(g(x)) = m$

$g(x) \mid x^n \oplus 1$ (it divides $x^n \oplus 1$)

$\Rightarrow g(x) \cdot h(x) = x^n \oplus 1$

$h(x)$: parity check polynomial

$\boxed{w(x) \cdot h(x) = 0}$

BINARY CYCLIC CODES

- a bit rotation of a code word yields again a code word

$\boxed{1}\boxed{0}\boxed{0}\boxed{1} \qquad \underbrace{\boxed{1}\boxed{1}\boxed{0}\boxed{0}}_{w}$

Can be represented as polynomials of degree $n-1$

$\Rightarrow$ all operations performed in $\boxed{\mathrm{mod}\ x^n \oplus 1}$

$\vec{w} = (1001) \Rightarrow w(x) = 1 \oplus x^3$

$\vec{w}' = (0110) \Rightarrow w'(x) = x \oplus x^2$

Ex: $w = (1001) \Rightarrow w(x) = 1 \oplus x^3$

rotate one bit to the right:

$w'(x) = x \cdot w(x) = x \oplus x^4 \ \underline{\mathrm{mod}}\ x^4 \oplus 1$

$= 1 \oplus x$

$$\begin{array}{l} x^4 \oplus x : x^4 \oplus 1 = 1 \\ x^4 \oplus 1 \\ \hline x \oplus 1 \end{array}$$

$\leftarrow$ not interesting for us

but this is

rotate 2 bits:

$w''(x) = x^2 \cdot w(x) = x^2 \oplus x^5 \ \mathrm{mod}\ x^4 \oplus 1$

$= x \oplus x^2$

$$\begin{array}{l} x^5 \oplus x^2 : x^4 \oplus 1 = \underline{x} \\ x^5 \oplus x \\ \hline x^2 \oplus x \end{array}$$

Ex: $w = (1001)$ is a cyclic code

other code word are for sure: (0011)

(0110)

(1100)

$$\begin{array}{r} 001001 \\ \oplus\, 010001 \\ \hline 011000 \end{array}$$