Spectrum of periodical signals (Fourier analysis and synthesis)

Signals and codes (SK)

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Exercise 2

Exercise content

- Computing spectrum of periodical signals using Fourier series
  - Fourier analysis
  - Fourier synthesis
  - Plotting the spectrum
  - Influence of sampling

Exercises

Exercise 02_1: Spectrum of a signal composed of sinusoids

Consider following continuous time signal with fundamental frequency f_0 = 100 Hz

$$x(t) = 4 + 4\cos(2\pi \cdot f_0\, t) + 3\cos\left(2\pi \cdot 2f_0 t + \frac{\pi}{4}\right) + 3\sin(2\pi \cdot 3f_0 t) + 2.5\cos\left(2\pi \cdot 5f_0 t - \frac{\pi}{4}\right)$$

a) Perform Fourier analysis to obtain Fourier coefficients {ak} from signal x(t)

b) Perform Fourier synthesis to obtain signal x2(t) from Fourier coefficients {ak}

c) Create MATLAB script that plots the following 4 plots adjacently

1. Original signal x(t).
2. Magnitudes of Fourier coefficients {ak} (i.e. Magnitude spectrum)
3. Phases of Fourier coefficients {ak} (i.e. Phase spectrum)
4. Synthesized signal x2(t)

d) Compare the results to the spectrum computed by hand using inverse Euler formulas

e) Observe what happens, if the signal is not sufficiently sampled

```
Help: use figure('Position', [100, 100, 1300, 500]); %defining position of corners of the figure
subplot(1,4,2) %defining the matrix of plots – 1 row and 4 columns, 2nd plot will apply
stem([-n*f0:f0:0 f0:f0:n*f0] , ak_abs); %for active plot this will be drawn
```

Exercises

Exercise 02_2: Spectrum of the rectangular signal with parametric duty cycle

(duty cycle in Czech: *střída*)

Consider continuous time signal with fundamental period T_0 = 10 ms defined as

$$x(t) = \begin{cases} 1 \ldots 0 \leq t < duty\_cycle \cdot T_0 \\ 0 \ldots duty\_cycle \cdot T_0 \leq t < T_0 \end{cases}$$

The values of *duty_cycle* are considered within interval < 0 , 1 >.

a) Solve the subtasks a) to c) from the first exercise by modifying the respective Matlab code. Consider first 10 harmonics.

b) Start with *duty_cycle* = 0.5 and compare the results with lecture 03, Ex.3_8

c) Observe the results for the following values of *duty_cycle*

   a) *duty_cycle* = 0 vs. *duty_cycle* = 1

   b) *duty_cycle* = 0.1 vs. *duty_cycle* = 0.9

   c) *duty_cycle* = 0.2 vs. *duty_cycle* = 0.8

Exercises

Exercise 02_3: Spectrum of the rectangular signal with fixed t_{on} and increasing t_{off}

Consider continuous time signal with fundamental period T_0 = 50 ms defined as

$$x(t) = \begin{cases} 1 \ldots 0.00 \le t < 0.01 \text{ s} \\ 0 \ldots 0.01 \le t < 0.05 \text{ s} \end{cases}$$

The values of *duty_cycle* are considered within interval < 0 , 1 >.

a) Solve the subtasks a) to c) from the first exercise by modifying the respective Matlab code. Consider 20 harmonics.

b) Perform Fourier analysis and synthesis with a modification: compute $T_0 \cdot \{a_k\}$ instead of $\{a_k\}$ alone. When you synthesize the signal, multiply by $\frac{1}{T_0}$. Results should have the same shape, just different magnitudes.

c) Now let the same $t_{\text{on}} = 0.01$ s and increase t_{off} from 0.04 s to 0.09. Modify the number of considered harmonics like n=round(n*toff/0.05);

d) Do the same with $t_{\text{off}} = 0.19$ s. You should see further spectrum densification.

e) Imagine $t_{\text{off}} \to \infty$, you would obtain spectrum of nonperiodic rectangular pulse and the formula for Fourier series $T_0\{a_k\} = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t)\mathrm{e}^{-j2\pi f_0 k t}\mathrm{d}t$ will change into

Fourier transform $\{F(f)\} = \int_{-\infty}^{+\infty} f(t)\mathrm{e}^{-j2\pi f t}\mathrm{d}t$

Exercises

Exercise 02_4: Spectrum of the unknown measured data

Consider the following measured data acquired with the sample frequency fs = 2.5 kHz:

x=[15,17.163,21.501,23.556,20.713,14.635,9.7365,9.1036,11.653,13.164,10.225,3.4337,-3.1257,-5.6373,-4.0616,-2.1353,-3.8667,-9.7796,-16.393,-19.422,-17.725,-14.339,-13.635,-17.243,-22.521,-25,-22.521,-17.243,-13.635,-14.339,-17.725,-19.422,-16.393,-9.7796,-3.8667,-2.1353,-4.0616,-5.6373,-3.1257,3.4337,10.225,13.164,11.653,9.1036,9.7365,14.635,20.713,23.556,21.501,17.163,15,17.163,21.501,23.556,20.713,14.635,9.7365,9.1036,11.653,13.164,10.225,3.4337,-3.1257,-5.6373,-4.0616,-2.1353,-3.8667,-9.7796,-16.393,-19.422,-17.725,-14.339,-13.635,-17.243,-22.521,-25,-22.521,-17.243,-13.635,-14.339,-17.725,-19.422,-16.393,-9.7796,-3.8667,-2.1353,-4.0616,-5.6373,-3.1257,3.4337,10.225,13.164,11.653,9.1036,9.7365,14.635,20.713,23.556,21.501,17.163,15,17.163,21.501,23.556,20.713,14.635,9.7365,9.1036,11.653,13.164,10.225,3.4337,-3.1257,-5.6373,-4.0616,-2.1353,-3.8667,-9.7796,-16.393,-19.422,-17.725,-14.339,-13.635,-17.243,-22.521,-25,-22.521,-17.243,-13.635,-14.339,-17.725,-19.422,-16.393,-9.7796,-3.8667,-2.1353,-4.0616,-5.6373,-3.1257,3.4337,10.225,13.164,11.653,9.1036,9.7365,14.635,20.713,23.556,21.501,17.163]

a) Plot the measured data. How many fundamental periods you see?

b) Find the spectrum of the signal.

c) What happens if you would consider first 50 harmonics?