Spectrum of periodical signals (Fourier analysis and synthesis)

Signals and codes (SK)

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Exercise 2

Exercise content

- Computing spectrum of periodical signals using Fourier series
  - Fourier analysis
  - Fourier synthesis
  - Plotting the spectrum
  - Influence of sampling

Exercises

Exercise 02_1: Spectrum of a signal composed of sinusoids

Consider following continuous time signal with fundamental frequency f_0 = 100 Hz

$$x(t) = 4 + 4\cos(2\pi \cdot f_0\, t) + 3\cos\left(2\pi \cdot 2f_0 t + \frac{\pi}{4}\right) + 3\sin(2\pi \cdot 3f_0 t) + 2.5\cos\left(2\pi \cdot 5f_0 t - \frac{\pi}{4}\right)$$

a) Perform Fourier analysis to obtain Fourier coefficients {*ak*} from signal *x*(*t*)

b) Perform Fourier synthesis to obtain signal *x2*(*t*) from Fourier coefficients {*ak*}

c) Create MATLAB script that plots the following 4 plots adjacently

1. Original signal *x*(*t*).
2. Magnitudes of Fourier coefficients {*ak*} (i.e. Magnitude spectrum)
3. Phases of Fourier coefficients {*ak*} (i.e. Phase spectrum)
4. Synthesized signal *x2*(*t*)

d) Compare the results to the spectrum computed by hand using inverse Euler formulas

e) Observe what happens, if the signal is not sufficiently sampled

```
%% defining parameters

n=5; % n>0, number of harmonics of Fourier series to approximate signal.
f0=100; %fundamental frequency
fs=???*f0; %sample frequency
```

Help:

1) `figure('Position', [100, 100, 1300, 500]);`
`%makes new figure defining position of its corners in brackets`

2)
```
subplot(1,4,1) %defining the matrix of plots of 1 row and 4 columns, 1st plot is active
plot(t,x); %to be drawn for active subplot
subplot(1,4,2) %defining the matrix of plots of 1 row and 4 columns, 2nd plot is active
stem(k*f0,ak_abs); %to be drawn for active subplot
```

Hints for implementing Fourier analysis:
- declare `k=-n:n;`
- declare `ak=zeros(1,length(k));`
- use `for` cycle to compute `ak` for each `k`

```
for i=1:length(k)
    ak(i)=???;
end
```

Exercises

Exercise 02_2: Spectrum of the rectangular signal with parametric duty cycle (duty cycle in Czech: ***střída***)

Consider continuous time signal with fundamental period T_0 = 10 ms defined as

$$x(t) = \begin{cases} 1 \ldots 0 \le t < duty\_cycle \cdot T_0 \\ 0 \ldots duty\_cycle \cdot T_0 \le t < T_0 \end{cases}$$

The values of *duty_cycle* are considered within interval < 0 , 1 >.

a) Solve the subtasks a) to c) from the first exercise by modifying the respective Matlab code. Consider first 10 harmonics.

b) Start with `duty_cycle = 0.5;` and compare the results with lecture 03, Ex.3_8

c) Observe the results for the following values of `duty_cycle`

   a) `duty_cycle = 0;` vs. `duty_cycle = 1;`

   b) `duty_cycle = 0.1;` vs. `duty_cycle = 0.9;`

   c) `duty_cycle = 0.2;` vs. `duty_cycle = 0.8;`

```
%% defining parameters

n=10; %n>0, number of harmonics of Fourier series to
approximate signal.
duty=0.5; % duty cycle of the rectangular signal
f0=100; %fundamental frequency
fs=???*f0; %sample frequency
```

Hint: defining rectangular signal
`x=zeros(1,length(t)); %start with zeros`
then overwrite „first part“ of x with ones

Exercises

Exercise 02_3: Spectrum of the rectangular signal with fixed t_{on} and increasing t_{off}

Consider continuous time signal with fundamental period $T_0 = t_{on} + t_{off} = 50$ ms defined as

$$x(t) = \begin{cases} 1 \ldots 0.00 \leq t < 0.01 \text{ s} \\ 0 \ldots 0.01 \leq t < 0.05 \text{ s} \end{cases}$$

The values of *duty_cycle* are considered within interval < 0 , 1 >.

a) Solve the subtasks a) to c) from the previous exercise by modifying the respective Matlab code. Consider 20 harmonics.

b) Perform Fourier analysis and synthesis with a modification: compute $T_0 \cdot \{a_k\}$ instead of $\{a_k\}$ alone. When you synthesize the signal, multiply by $\frac{1}{T_0}$. Results should have the same shape, but different magnitudes.

c) Now let the same $t_{on} = 0.01$ s and increase t_{off} from 0.04 s to 0.09. Modify the number of considered harmonics like `n=round(n*T0/0.05);`

d) Do the same with $t_{off} = 0.19$ s. You should see further spectrum densification.

e) Note: now imagine $t_{off} \to \infty$, you would obtain spectrum of nonperiodic rectangular pulse and the formula for Fourier series $T_0\{a_k\} = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t)\mathrm{e}^{-j2\pi f_0 kt}\mathrm{d}t$ will change into

Fourier transform $\{F(f)\} = \int_{-\infty}^{+\infty} f(t)\mathrm{e}^{-j2\pi f t}\mathrm{d}t$

```
%% defining parameters

ton=0.01; % first part of rectangle waveform - time of ones
toff=0.04; % second part of rectangle waveform - time of zeros
fs=1e6;
n=20; %N>0, number of spectral lines on each side for original waveform with toff=0.04.
```

Exercises

Exercise 02_4: Spectrum of the unknown measured data

Consider the following measured data acquired with the sample frequency fs = 2.5 kHz:

x=[15,17.163,21.501,23.556,20.713,14.635,9.7365,9.1036,11.653,13.164,10.225,3.4337,-3.1257,-5.6373,-4.0616,-2.1353,-3.8667,-9.7796,-16.393,-19.422,-17.725,-14.339,-13.635,-17.243,-22.521,-25,-22.521,-17.243,-13.635,-14.339,-17.725,-19.422,-16.393,-9.7796,-3.8667,-2.1353,-4.0616,-5.6373,-3.1257,3.4337,10.225,13.164,11.653,9.1036,9.7365,14.635,20.713,23.556,21.501,17.163,15,17.163,21.501,23.556,20.713,14.635,9.7365,9.1036,11.653,13.164,10.225,3.4337,-3.1257,-5.6373,-4.0616,-2.1353,-3.8667,-9.7796,-16.393,-19.422,-17.725,-14.339,-13.635,-17.243,-22.521,-25,-22.521,-17.243,-13.635,-14.339,-17.725,-19.422,-16.393,-9.7796,-3.8667,-2.1353,-4.0616,-5.6373,-3.1257,3.4337,10.225,13.164,11.653,9.1036,9.7365,14.635,20.713,23.556,21.501,17.163,15,17.163,21.501,23.556,20.713,14.635,9.7365,9.1036,11.653,13.164,10.225,3.4337,-3.1257,-5.6373,-4.0616,-2.1353,-3.8667,-9.7796,-16.393,-19.422,-17.725,-14.339,-13.635,-17.243,-22.521,-25,-22.521,-17.243,-13.635,-14.339,-17.725,-19.422,-16.393,-9.7796,-3.8667,-2.1353,-4.0616,-5.6373,-3.1257,3.4337,10.225,13.164,11.653,9.1036,9.7365,14.635,20.713,23.556,21.501,17.163]

a) Plot the measured data. How many fundamental periods do you observe?

b) Find the spectrum of the signal.

c) What happens if you would consider first 50 harmonics?

```
%% defining parameters

fs=2500;
```

Help:
after you have solved subtask a), you can
1) use
`x=x(1:end/noT); %noT...number of fundamental periods`
`t=t(1:end/noT);`
to reduce the size of `x` and `t` to one period only
2) then use the previous scripts to perform Fourier analysis and synthesis