20SK – Signals and Codes

Lecture 1 – Introduction to Digital Communications (2018/11/12)

Topics discussed:

- Digital communication systems, binary interface between source and channel
- Source coding/decoding and channel coding/decoding
- Reasons why communication systems now usually contain a binary interface between source and channel (i.e., why digital communication systems are now standard)
- Source coding
- Entropy of a discrete signal source, examples
- Communication channels
- Shannon's coding theorem
- Channel encoding (modulation)
- Role of Error correcting codes
- Digital interface

The relevant literature to study is [1, chapter 1].

Resources

[1] Gallager, R.: Course materials for 6.450 *Principles of Digital Communications I*, Fall 2006. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology.

Reasons for using binary interface

1. Shannon has proven that it is a good way to do so
2. Hardware is a) very cheap and b) small
3. Standardization
4. Simplifies implementation and understanding

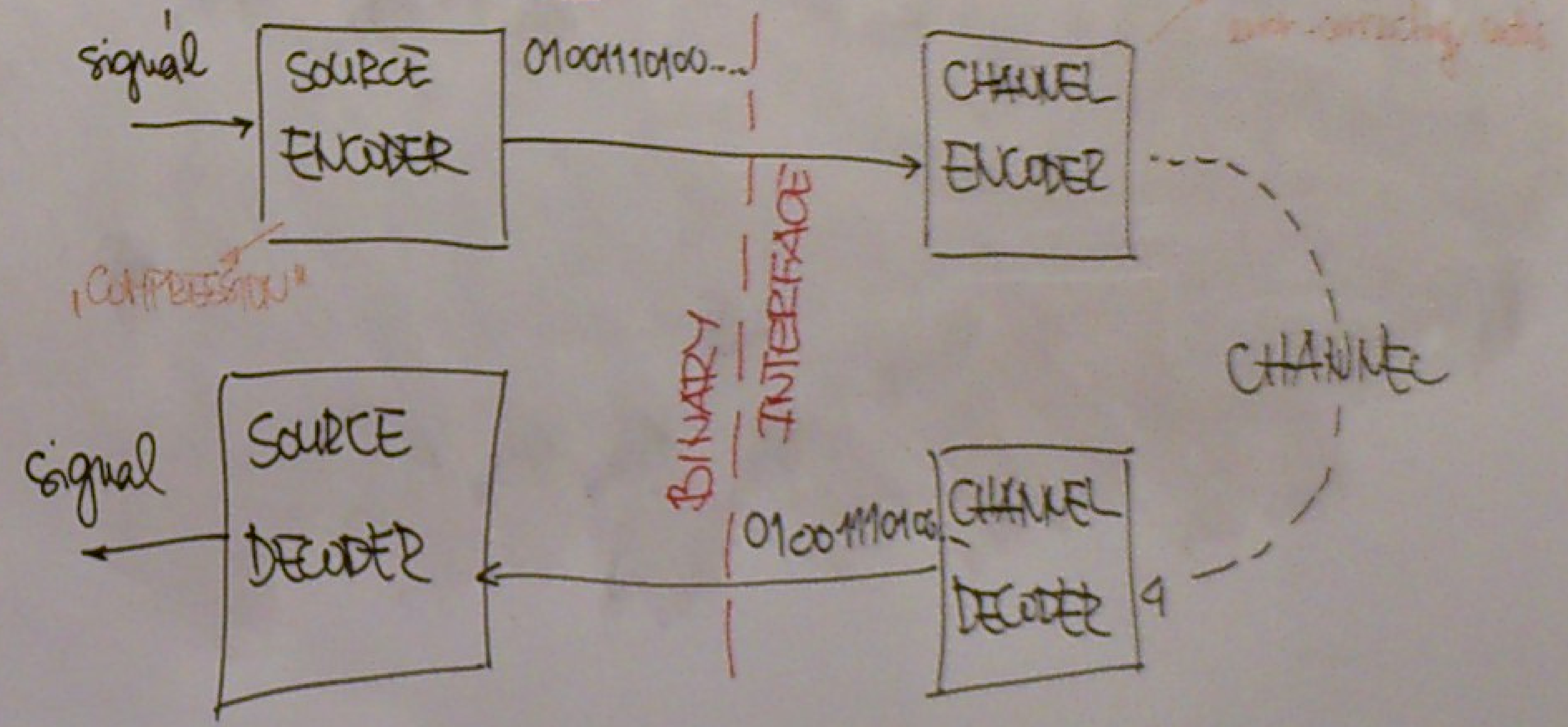
for single-source & single-reciever

Web postupně migruje:

http://zolotarev.fd.cvut.cz/ske/

Digitální komunikace

1948 Claude Shannon: vše lze převést jako 0,1

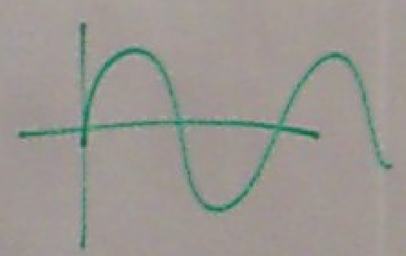


SOURCE ENCODING

Communication source

- Nyquist : superimposed sine wave ⇒ Fourier analysis
- Shannon : random process ⇒ $\boxed{\text{information theory}}$

Example: $f(t) = a \cdot \underline{\sin}(\omega t + \varphi)$



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↳ Shannon: (a, ω, φ)

$\boxed{\text{ENTROPY}}$ in bits

Random process generated by a source $X = \{x_1, x_2, x_3, x_4, \ldots, x_n\}$

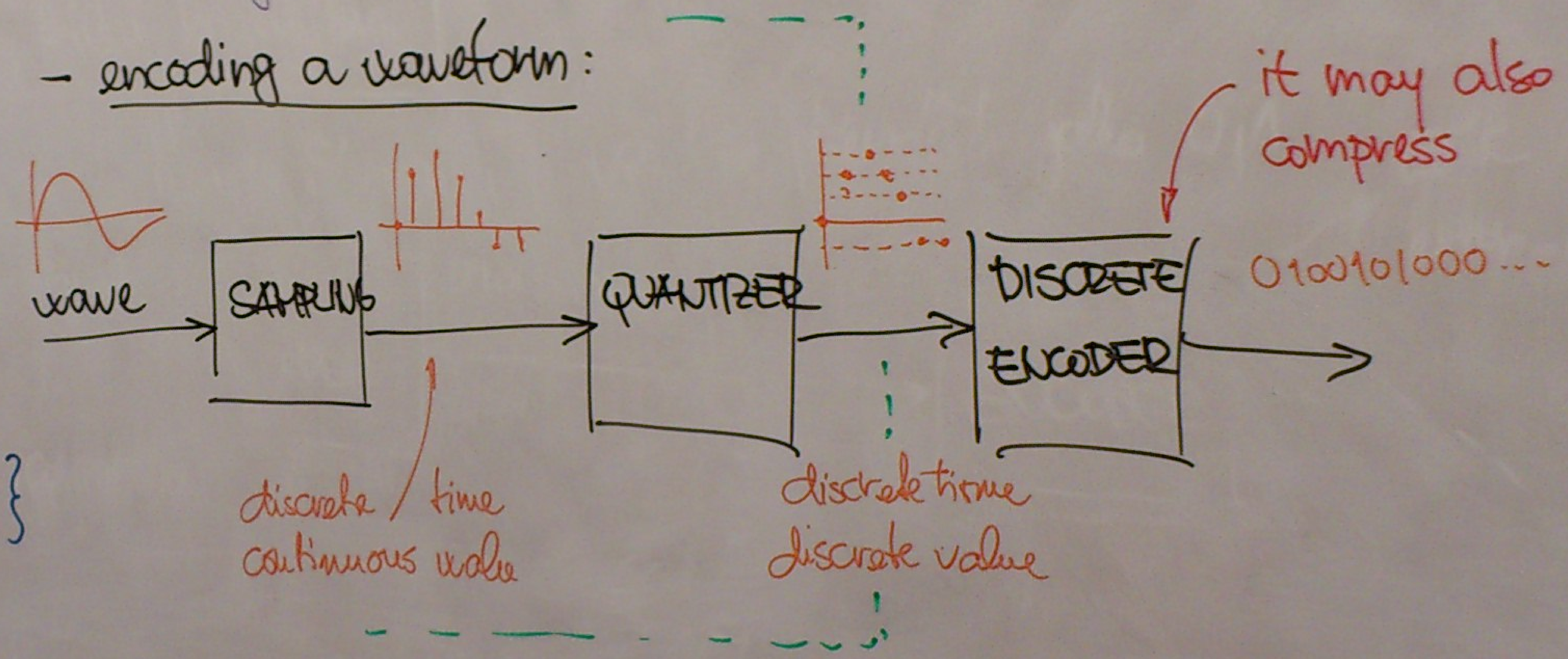
possible "symbol" generated by the source

with probabilities $p(x_1), p(x_2), p(x_3) \ldots$

$$H(X) = -\sum_{i=1}^{n} p(x_i) \cdot \log_2(p(x_i))$$

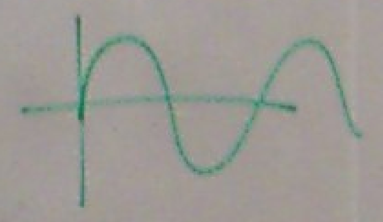
→ number of <u>bits</u> necessary to represent the source

- OSI network model
  - ↳ layers & interfaces
- <u>encoding a waveform</u> :



Communication source

- Nyquist : superimposed sine wave $\Rightarrow$ Fourier analysis
- Shannon : random process $\Rightarrow$ $\boxed{\text{information theory}}$

Example: $f(t) = a \cdot \underline{\sin}(\omega t + \varphi)$  .....

↳ Shannon: (a, ω, φ)

$\boxed{\text{ENTROPY}}$ in bits

Random process generatet by a souce $X = \{x_1, x_2, x_3, x_4, \dots, x_n\}$ possible "symbol" generated by the source

with probabilities $p(x_1), p(x_2), p(x_3) \dots$

$$H(X) = -\sum_{i=1}^{N} p(x_i) \cdot \log_2(p(x_i))$$

→ number of bits necessary to represent the source

Example: Fair dice $p(x_i) = 1/6$ $X = \{1, 2, 3, 4, 5, 6\}$

$$H(X) = -\sum_{i=1}^{6} \frac{1}{6} \log_2 \frac{1}{6} = -\log_2 \frac{1}{6} = -(-2.585)$$

↳ i need 3 bits to represent numbers 1 ... 6.

$\boxed{\text{Shannon's coding theorem}}$

Source with entropy $H(X)$ producing $\underline{r}$ symbols / sec

→ source speed : $\boxed{V(X) = H(X) \cdot r}$ [bps]

Communication channel with capacity C [bps]

a) $V(X) > C$ $\Rightarrow$ i can not transmit anything

b) $V(X) < C$ $\Rightarrow$ $V(X)$ can be transmitted with an arbitrary low error

c) $V(X) = C$ i camot make guarantees about the error

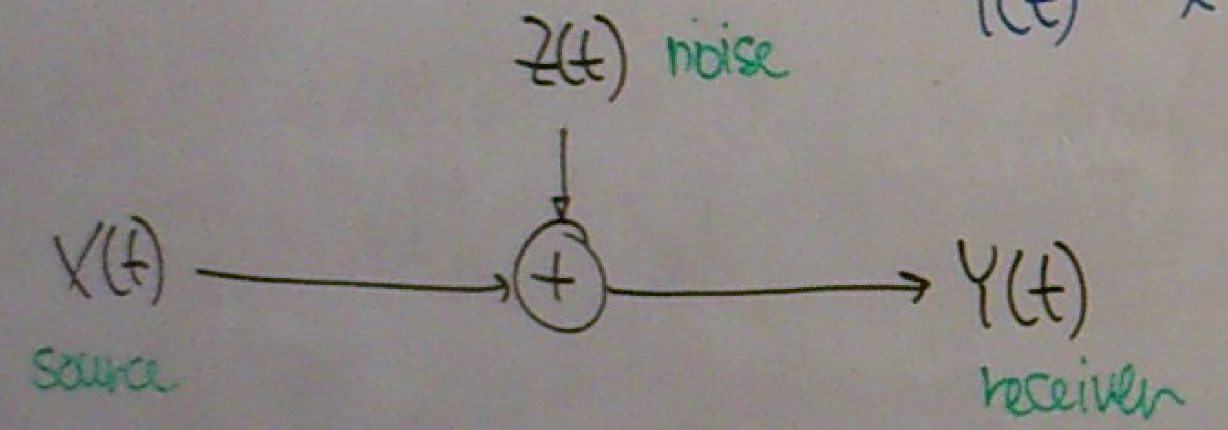
$\boxed{\text{CHANNEL CODING}}$

The channel is given; we have no influence over it

converting 0's and 1's $\longrightarrow$ modulation

main problem: $\boxed{\text{noise}}$

$$Y(t) = X(t) + Z(t)$$



AWGN channel (Additive white Gaussian noise)

Example: Fair dice $p(x_i) = 1/6$ $X = \{1,2,3,4,5,6\}$

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Source with entropy $H(X)$ producing $\underline{r}$ symbols / sec

$\rightarrow$ source speed : $\boxed{V(x) = H(x) \cdot r}$ [bps]

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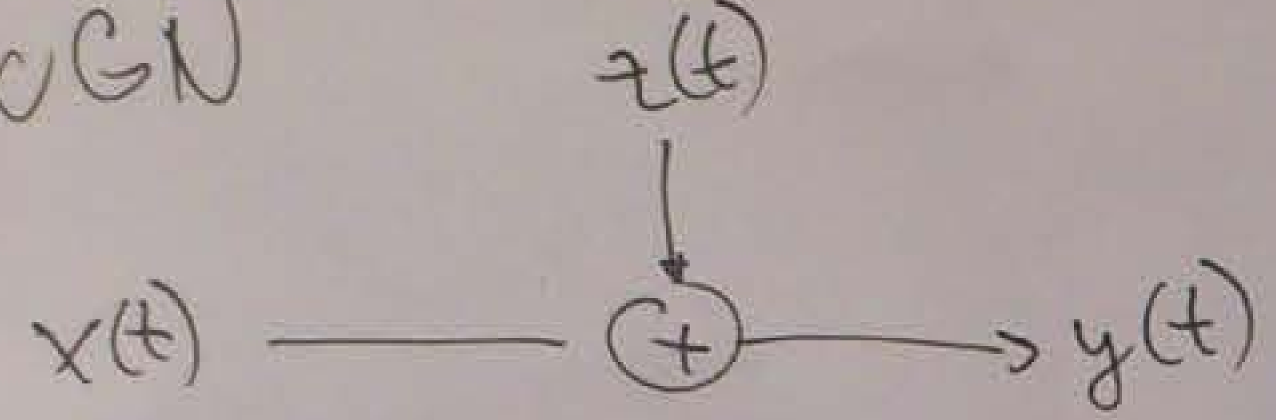
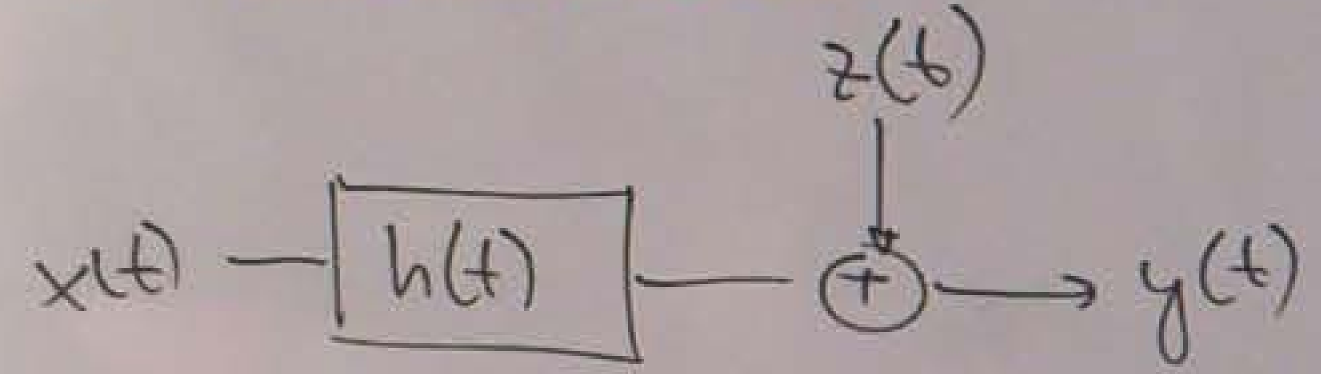
AWGN



$$y(t) = x(t) + z(t)$$

→ linear Gaussian channel



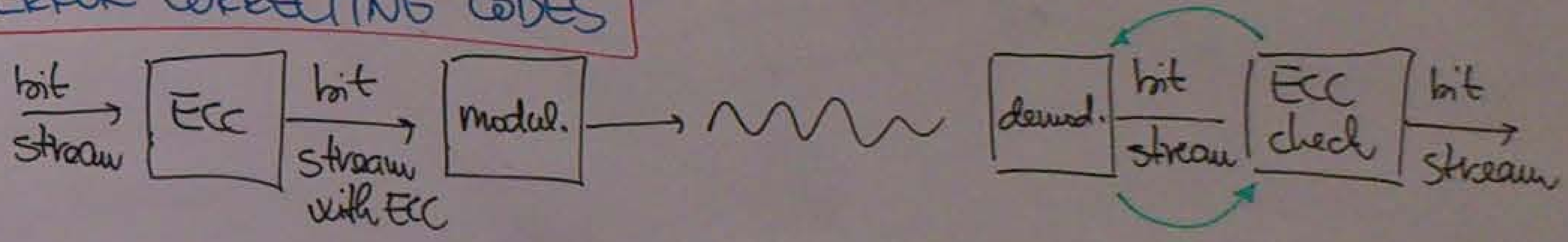
$$\boxed{y(t) = x(t) * h(t) + z(t)}$$

- wired
- wireless (only for direct line-of-sight transmitter & receiver static)

CHANNEL ENCODING

- encoded signal is stream of 0s and 1s
  - → modulation (PAM, PSK, QPSK, QAM, ...)
  - → convert signal to complex number → waveform ⊕ noise
  - → different waveform of the receiver
  - → demodulation main task: detect the correct transmitted waveform
    - ↳ ERRORS !!

⇒ ERROR CORRECTING CODES



Shannon: A more sophisticated encoding scheme can achieve arbitrarily low error rates at any data rate below channel capacity

$$\boxed{C = W \cdot \log_2\left(1 + \frac{P}{N_0 W}\right) \text{ [bps]}}$$

only for AGWN proven

W [Hz] ... bandwidth of the channel

P [W] ... input power

N_0 [W/Hz] ... noise per unit bandwidth

Digital interfaces – complicating factors

- unequal data rates: rate of the source encoder $\neq$ input rate of channel encoder
- errors: source decoder needs an exact replica of the encoded data, but not always the channel decoder is able to provide it
- networks: different paths to destination, shared medium